Granularity of Conflicts and Dependencies in Graph Transformation Systems: Extended Version

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Abstract. Conflict and dependency analysis (CDA) is a static analysis for the detection of conflicting and dependent rule applications in a graph transformation system. The state-of-the-art CDA technique, critical pair analysis, provides its users the benefits of completeness, i.e., its output contains a precise representation of each potential conflict and dependency in a minimal context, called *critical pair*. Yet, user feedback has shown that critical pairs can be hard to understand; users are interested in core information about conflicts and dependencies occurring in various combinations. In this paper, we investigate the granularity of conflicts and dependencies in graph transformation systems. We introduce a variety of new concepts on different granularity levels: We start with *conflict atoms*, representing individual graph elements as smallest building bricks that may cause a conflict. We show that each conflict atom can be extended to at least one *conflict reason* and, conversely, each conflict reason is covered by atoms. Moreover, we relate conflict atoms to *minimal conflict reasons*, representing smallest element sets to be overlapped in order to obtain a pair of conflicting transformations. We show how conflict reasons are related to critical pairs. Finally, we introduce dual concepts for dependency analysis. As we discuss in a running example, our concepts pave the way for an improved CDA technique.

1 Introduction

Graph transformation systems (GTSs) are a fundamental modeling concept with applications in a wide range of domains, including software engineering, mechanical engineering, and chemistry. A GTS comprises a set of transformation rules that are applied in coordination to achieve a higher-level goal. The order of rule applications can either be specified explicitly using a control flow mechanism, or it is given implicitly by causal dependencies of rule applications. In the latter case, conflicts and dependencies affect the control flow. For instance, a rule may delete an element whose existence is required by another rule to modify the graph.

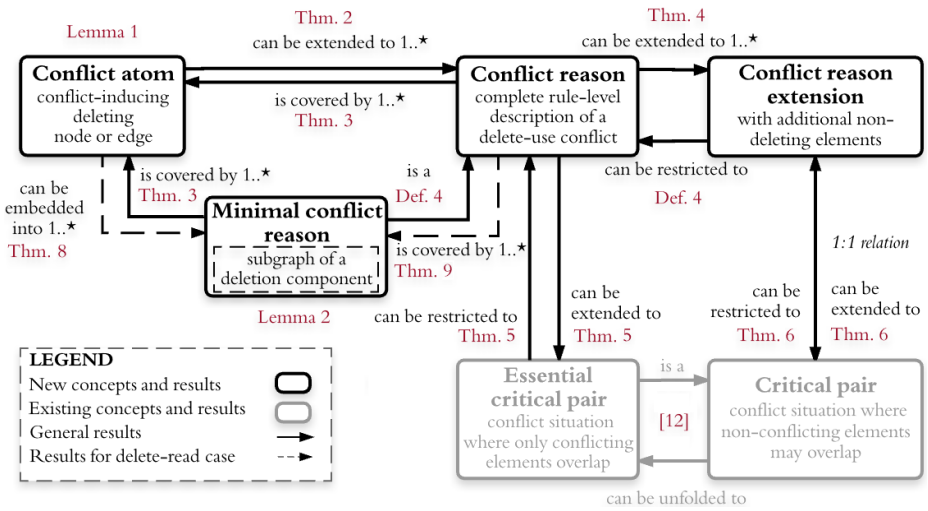


Fig. 1. Inter-relations of new and existing conflict notions

To better understand the implicit control flow of a GTS, one needs to analyze the potential conflicts and dependencies of its rule applications. Conflict and dependency analysis (CDA) is a static analysis for the detection of such conflicts and dependencies. An important CDA technique is *critical pair analysis* [1, 2], which has been used in the literature to detect conflicting functional requirements [3], feature interactions [4], conflicting and dependent change operations for process models [5], causal dependencies of aspects in aspect modeling [6], potential conflicts and dependencies between refactorings [7, 8], and to validate service-oriented architectures [9].

In these applications, there are generally two possible usage scenarios for CDA: First, the user may start with a list of expected conflicts and dependencies that are supposed to occur. CDA is then used to determine if the expected conflicts and dependencies in fact arise, and/or if there are any unexpected conflicts and dependencies. Violations of expectations signify potential errors in the rule specifications, and can be used for debugging [10]. Second, the user may want to improve their transformation system to reduce conflicts and dependencies, so that rules can be applied independently, e.g., to enable a collaborative modeling process based on edit operation rules [11]. In this case, conflicts and dependencies reported by CDA can be used to identify required modifications. In both cases, users need to inspect conflicts or dependencies to pinpoint their root causes.

To support users during this task, in this work, we lay the basis for a refined CDA technique, distinguishing a variety of new concepts to describe conflicts and dependencies between rules. Our investigation is guided by the notion of *granularity*, and, building on the existing theory for algebraic graph transformation, focuses on *delete-use*-conflicts. We introduce a variety of new concepts and their relations as summarized in Fig. 1. First, we introduce *conflict atoms*, i.e., single graph elements causing conflicts, to represent smallest entities of con-

flicts. Each conflict atom can be embedded into the *conflict reason* of a pair of conflicting rules, while each such conflict reason is fully covered by conflict atoms. A conflict reason comprises all elements being deleted by the first (or being boundary nodes) and required by the second rule of the considered rule pair. Conflict reasons correspond to essential critical pairs as introduced in previous work [12]. A special type of conflict reasons are *minimal conflict reasons*, representing conflicting graphs and embeddings that are minimal in the sense that they comprise smallest sets of elements required to yield a valid pair of conflicting transformations. Fourth and finally, conflict reasons can be augmented to conflict reason extensions, which have a one-to-one relationship with the notion of critical pairs [1]. Conflict atoms and minimal conflict reasons are more coarse-grained in the sense that they generally represent a larger number of potential conflicts while abstracting away many details of these conflicts, whereas conflict reasons and conflict reason extensions are more fine-grained since they describe conflicts more precisely.

For the relationships between conflict atoms, regular conflict reasons, conflict reason extensions, and the existing notions of critical pairs, we provide full formal characterizations. In the case of minimal conflict reasons and their relationships, we restrict ourselves to a special case, in which the second rule in the considered rule pair is non-deleting. In other words, we only consider *delete-read*-conflicts, while abstracting from *delete-delete*-conflicts. To still use the concept of minimal conflict reasons in situations where the second rule actually performs deletions, one can consider a non-deleting variant of the second rule, which allows an over-approximation of minimal conflict reasons.

With this contribution, we aim to improve on the state-of-the-art CDA technique, critical pair analysis (CPA) [1, 2], by offering improved support for cases where the CPA results did not match the user expectations. In CPA, all potential conflicts and dependencies that can occur when applying two rules are displayed in a minimal context. Confidence in CPA is established by positive fundamental results: via the Completeness Theorem, there exists a critical pair for each conflict, representing this conflict in a minimal context. However, experiences with the CPA indicate two drawbacks: (i) understanding the identified critical pairs can be a challenging task since they display too much information (i.e., they are too fine-grained), (ii) calculating the results can be computationally expensive. Our investigation provides the basis for a solution to compute and report potential conflicts on a level of detail being suitable for the task at hand.

This paper is an extended version of [13], in which we first investigated the granularity of conflict and dependencies in GTSs. In the present paper, we provide proofs of all formal results from [13]. In addition, we provide an exact characterization of minimal conflict reasons based on a certain overapproximation: we consider a non-deleting variant of the second rule in the considered rule pair. Specifically, we make three contributions.

- We present a conceptual consideration of conflicts in GTSs, based on the notion of granularity, and focusing on delete-use-conflicts.
- We introduce a variety of formal results for interrelating the new concepts with each other and with the existing concepts. In particular, we relate the

new concepts to the well-known conflict concepts of essential and regular critical pairs and characterize minimal conflict reasons for delete-read conflicts.
- We discuss how these concepts and results can be transferred to dependencies in a straight-forward manner. In particular, we introduce dependency atoms and reasons, the dual concepts to those introduced for conflict analysis.

The rest of this paper is structured as follows: In Sect. 2, we recall graph transformation concepts and conflict notions from the literature. In Sect. 3, we present the new concepts and formal results. We compare with related work and conclude in Section 4. In addition, we present all proofs of new results in the appendix.

2 Preliminaries

As a prerequisite for our new analysis of conflicts and dependencies, we recall the double-pushout approach to graph transformation as presented in [2]. Furthermore, we reconsider two notions of conflicting transformation and their equivalence as shown in [12].

2.1 Graph Transformation: Double-Pushout Approach

Throughout this paper we consider graphs and graph morphisms as presented in [2]; since most of the definitions and results are given in a category-theoretical way, the extension to e.g. typed, attributed graphs [2] is prepared, but up to future work.[1]

Graph transformation is the rule-based modification of graphs. A *rule* mainly consists of two graphs: L is the left-hand side (LHS) of the rule representing a pattern that has to be found to apply the rule. After the rule application, a pattern equal to R, the right-hand side (RHS), has been created. The intersection $K = L \cap R$ is the graph part that is not changed, the graph part that is to be deleted is defined by $L \setminus (L \cap R)$, while $R \setminus (L \cap R)$ defines the graph part to be created. Throughout this paper we consider a graph transformation system just as a set of rules.

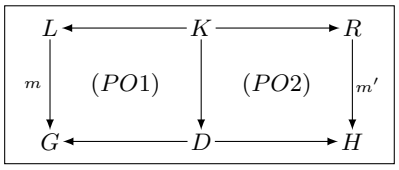
A *graph transformation step* $G \overset{m,r}{\Longrightarrow} H$ between two graphs G and H is defined by first finding a graph morphism[2] m of the LHS L of rule r into G such that m is injective, and second by constructing H in two passes: (1) build $D := G \setminus m(L \setminus K))$, i.e., erase all graph elements that are to be deleted; (2) construct $H := D \cup m'(R \setminus K)$ such that a new copy of all graph elements that are

[1] As for the notion of essential critical pairs [12] in categorical terms we would need pullbacks and pushouts over general morphisms (in case of non-injective matching) as well as initial pushouts over rule morphisms.

[2] A morphism between two graphs consists of two mappings between their nodes and edges being both structure-preserving w.r.t. source and target functions. Note that we denote inclusions by $\hookrightarrow$ and all other morphisms by $\rightarrow$.

to be created is added. It has been shown for graphs and graph transformations that r is applicable at m iff m fulfills the *gluing condition* [2]. In that case, m is called a *match*. For injective morphisms as we use them here, the gluing condition reduces to the *dangling condition*. It is satisfied if all adjacent graph edges of a graph node to be deleted are deleted as well, such that D becomes a graph. Injective matches are usually sufficient in applications and w.r.t. our work here, they allow to explain constructions much easier than for general matches.

Definition 1 (Rule and transformation). *A* rule r *is defined by* $r = (L \hookleftarrow K \hookrightarrow R)$ *with* L, K, *and* R *being graphs connected by two graph inclusions. A* (direct) transformation $G \overset{m,r}{\Longrightarrow} H$ *which applies rule* r *to a graph* G *consists of two pushouts as depicted below. Morphism* $m : L \to G$ *is injective and is called* match. *Rule* r *is* non-deleting *if* $L = K$. *Rule* r *is* applicable *at match* m *if there exists a graph* D *such that* $(PO1)$ *is a pushout.*



Example 1. Refactoring is a generally acknowledged technique to improve the design of an object-oriented system [14]. To achieve a larger improvement there is typically a sequence of refactorings required. Due to implicit conflicts and dependencies that may occur between refactorings, it is not always easy for developers to determine which refactorings to use and in which order to apply them. To this aim, CDA can support the developer in finding out if there are conflicts or dependencies at all and, if this is the case, in understanding them.

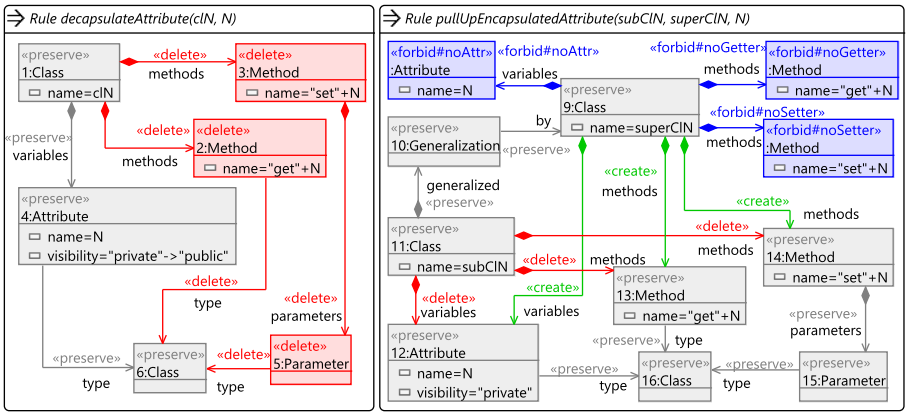


Fig. 2. Refactoring rules *decapsulateAttribute* and *pullUpEncapsulatedAttribute*.

Assuming graphs that model the class design of software systems, we consider Fig. 2 for two class model refactorings being specified as graph-based transformation rules. Rules are depicted in an integrated form where annotations specify

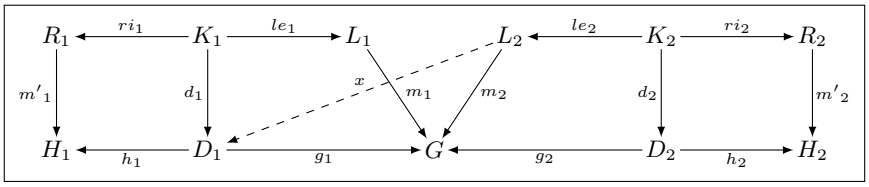
which graph elements are deleted, preserved, and created. While the preserved and deleted elements form the LHS of a rule, the preserved and created elements form its RHS. Moreover, negative application conditions specify graph elements that are forbidden when applying a rule. Rule *decapsulateAttribute* removes getter and setter methods for a given attribute, thus inverting the well-known *encapsulate attribute* refactoring. Rule *pullUpEncapsulatedAttribute* takes an attribute with its getter and setter methods and moves them to a superclass if there are not already equally named elements.

2.2 Conflicting Transformations

In this subsection, we recall the essence of conflicting transformations. We concentrate on delete-use conflicts which means that the first rule application deletes graph items that are used by the second rule application. In the literature, there are two different definitions for delete-use conflicts. We recall these definitions and a theorem which shows the equivalence between these two.

The first definition [2] of a delete-use conflict states that the match for the second transformation cannot be found anymore after applying the first transformation. Note that we do not consider delete-use conflicts of the second transformation on the first one explicitly. To get also those ones, we simply consider the inverse pair of transformations. Note furthermore that delete-use conflicts degenerate to delete-read conflicts if rule r_2 is non-deleting.

Definition 2 (Delete-use conflict). *Given a pair of direct transformations* $(t_1, t_2) = (H_1 \overset{m_1,r_1}{\Longleftarrow} G \overset{m_2,r_2}{\Longrightarrow} H_2)$ *applying rules* $r_1 : L_1 \overset{le_1}{\hookleftarrow} K_1 \overset{ri_1}{\hookrightarrow} R_1$ *and* $r_2 : L_2 \overset{le_2}{\hookleftarrow} K_2 \overset{ri_2}{\hookrightarrow} R_2$ *as depicted below. Transformation* t_1 *causes a* delete-use conflict *on transformation* t_2 *if there does not exist a morphism* $x : L_2 \to D_1$ *such that* $g_1 \circ x = m_2$.



In the following, we consider an alternative characterization for a transformation to cause a delete-use conflict on another one (as introduced in [12]). It states that at least one deleted element of the first transformation is overlapped with some used element of the second transformation. This overlap is formally expressed by a span of graph morphisms between the minimal graph C_1, containing all elements to be deleted by the first rule, and the LHS of the second rule (Fig. 3). In particular, we use an initial pushout construction [2] over the left-hand side morphism of the rule to compute the *boundary graph* B_1 consisting of all nodes needed to make $L_1 \setminus K_1$ a graph and the *context graph* $C_1 := L_1 \setminus (K_1 \setminus B_1)$. We say that the nodes in B_1 are *boundary nodes*. Graph

fragment $C_1 \setminus B_1$ is the *deletion part* of rule r_1. It may consist of several disjoint fragments, called *deletion fragments*. Completing a deletion fragment to a graph by adding all incident nodes (i.e. boundary nodes) it becomes a *deletion component* in C_1. Each two deletion components overlap in boundary nodes only; the union of all deletion components is C_1. If two transformations overlap such that there is at least one element of a deletion fragment included, they are also conflicting. The equivalence of these two conflict notions is recalled in the following theorem.

Theorem 1 (Delete-use conflict characterization). *Given a pair of transformations* $(t_1, t_2) = (H_1 \overset{m_1,r_1}{\Longleftarrow} G \overset{m_2,r_2}{\Longrightarrow} H_2)$ *via rules* $r_1 : L_1 \overset{le_1}{\hookleftarrow} K_1 \overset{ri_1}{\hookrightarrow} R_1$ *and* $r_2 : L_2 \overset{le_2}{\hookleftarrow} K_2 \overset{ri_2}{\hookrightarrow} R_2$*, the initial pushout (1) for* $K_1 \overset{le_1}{\hookrightarrow} L_1$*, and the pullback (2) of* $(m_1 \circ c_1, m_2)$ *in Fig. 2 yielding the span* $s_1 : C_1 \overset{o_1}{\hookleftarrow} S_1 \overset{q_{12}}{\Rightarrow} L_2$*, then the following equivalence holds:* t_1 *causes a delete-use conflict on* t_2 *according to Def. 2 iff* $s_1 : C_1 \overset{o_1}{\hookleftarrow} S_1 \overset{q_{12}}{\Rightarrow} L_2$ *satisfies the* conflict condition *i.e. there does not exist any morphism* $x : S_1 \to B_1$ *such that* $b_1 \circ x = o_1$.

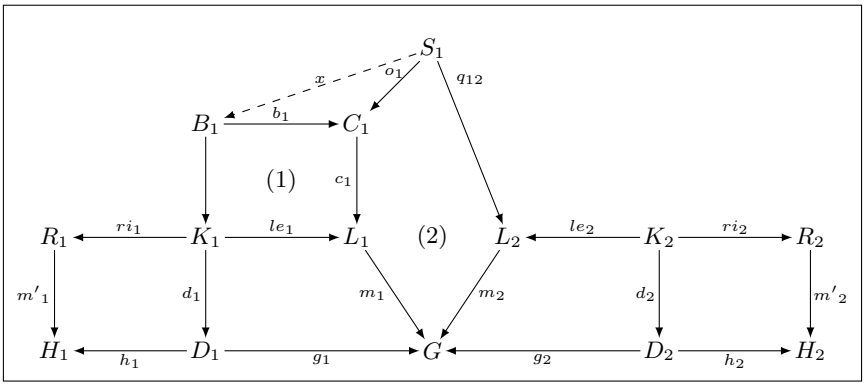


Fig. 3. Delete-use conflict characterization for transformations

In the rest of the paper we merely consider delete-use conflicts such that in the following we abbreviate *delete-use conflict* with *conflict*.

3 The Granularity of Conflicts and Dependencies

So far, a conflict between two transformations has always been considered as a whole. In the following, we investigate new notions of conflicting rules presenting them on different levels of granularity. Our intention is the possibility to gradually introduce users to conflicts. Starting with a coarse-grained conflict description in the form of conflict atoms, more information is gradually added until we arrive at the fine-grained representation of conflicts by critical pairs (as e.g. presented in [2]), representing each pair of conflicting transformations in a

minimal context. Following this path we introduce several new concepts for conflicting rules and show their interrelations as well as their relations to (essential) critical pairs. Finally, we sketch dual concepts for dependencies.

3.1 Conflicting Rules: Considering Different Granularity Levels

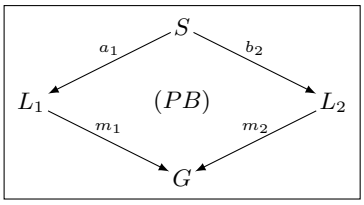
Now, we lift our conflict considerations from transformations to the rule level, i.e., we consider conflicting rules. Two rules are in conflict if there is a pair of conflicting transformations applying these rules. According to Theorem 1 there is a span between these rules specifying the conflict reasons or at least parts of it. In the following, we will concentrate on these spans and distinguish several forms of spans showing conflict reasons in different granularity.

We start focusing on minimal building bricks, called conflict atoms. In particular, we consider a conflict atom to be a minimal sub-graph of C_1 which can be embedded into L_2 but not into B_1 (*conflict and minimality conditions*). Moreover, a pair of direct transformations needs to exist for which the match morphisms overlap on the conflict atom (*transformation condition*). Note that, in general, the matches of this pair of transformations may overlap also in graph elements not contained in the conflict atom. Hence, such a a pair of transformations may be chosen flexibly, it need not show a conflict in a minimal context as critical pairs do. While conflict atoms describe the smallest conflict parts, a conflict reason is a complete conflict part in the sense that all in the reported conflict involved atoms are subsumed by it (*completeness condition*). While conflict reasons overlap in conflicting graph elements and boundary nodes only, conflict reason extensions may overlap in non-conflicting elements of the LHSs of participating rules as well (*extended completeness condition*).

Definition 3 (Basic conflict conditions). *Given rules* $r_1 : L_1 \overset{le_1}{\hookleftarrow} K_1 \overset{ri_1}{\hookrightarrow} R_1$ *and* $r_2 : L_2 \overset{le_2}{\hookleftarrow} K_2 \overset{ri_2}{\hookrightarrow} R_2$ *with the initial pushout (1) for* $K_1 \overset{le_1}{\hookrightarrow} L_1$ *as well as a span* $s_1 : C_1 \overset{o_1}{\hookleftarrow} S_1 \overset{q_{12}}{\to} L_2$ *as depicted in Fig. 3,* basic conflict conditions *for the span* s_1 *of* (r_1, r_2) *are defined as follows:*

1. Conflict condition*: Span* s_1 *satisfies the* conflict condition *if there does not exist any injective morphism* $x : S_1 \to B_1$ *such that* $b_1 \circ x = o_1$.
2. Transformation condition*: Span* s_1 *satisfies the* transformation condition *if there is a pair of transformations* $(t_1, t_2) = (H_1 \overset{m_{1,r_1}}{\Longleftarrow} G \overset{m_{2,r_2}}{\Longrightarrow} H_2)$ *via* (r_1, r_2) *with* $m_1(c_1(o_1(S_1))) = m_2(q_{12}(S_1))$ *(i.e. (2) is commuting in Fig. 3).*
3. Completeness condition*: Span* s_1 *satisfies the* completeness condition *if there is a pair of transformations* $(t_1, t_2) = (H_1 \overset{m_{1,r_1}}{\Longleftarrow} G \overset{m_{2,r_2}}{\Longrightarrow} H_2)$ *via* (r_1, r_2) *such that (2) is the pullback of* $(m_1 \circ c_1, m_2)$ *in Fig. 3.*
4. Minimality condition*: A span* $s'_1 : C_1 \overset{o'_1}{\hookleftarrow} S'_1 \overset{q'_{12}}{\to} L_2$ *can be embedded into span* s_1 *if there is an injective morphism* $e : S'_1 \to S_1$*, called* embedding morphism*, such that* $o_1 \circ e = o'_1$ *and* $q_{12} \circ e = q'_{12}$*. If* e *is an isomorphism, then we say that the spans* s_1 *and* s'_1 *are* isomorphic*. (See (3) and (4) in Fig. 4.) Span* s_1 *satisfies the* minimality condition *w.r.t. a set* SP *of spans if any* $s'_1 \in SP$ *that can be embedded into* s_1 *is isomorphic to* s_1.

Finally, span $s : L_1 \overset{a_1}{\hookleftarrow} S \overset{b_2}{\rightarrow} L_2$ fulfills the *extended completeness condition* if there is a pair of transformations $(t_1, t_2) = (H_1 \overset{m_1,r_1}{\Longleftarrow} G \overset{m_2,r_2}{\Longrightarrow} H_2)$ via (r_1, r_2) such that s arises from the pullback of (m_1, m_2) in the figure on the right.



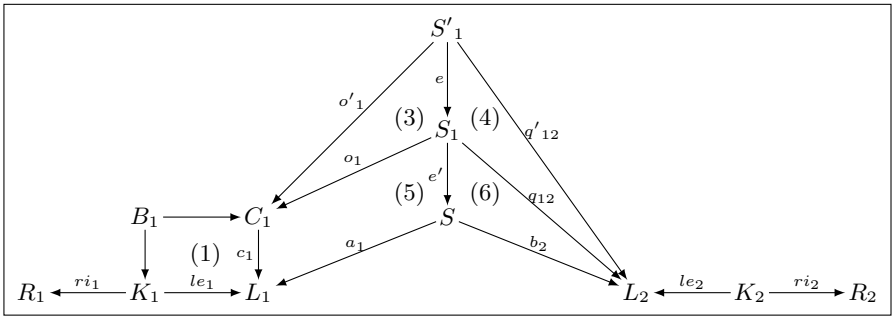


Fig. 4. Illustrating span embeddings

In the following, we define the building bricks of conflicts. The most basic notion to describe a conflict between two rules is that of a *conflict part*. Conflict parts may not describe the whole conflict between two rules. The smallest conflict parts are *conflict atoms*. If a conflict part describes a complete conflict, it is called *conflict reason*.

Definition 4 (Conflict notions for rules). *Let the rules* $r_1 : L_1 \overset{le_1}{\hookleftarrow} K_1 \overset{ri_1}{\hookrightarrow} R_1$ *and* $r_2 : L_2 \overset{le_2}{\hookleftarrow} K_2 \overset{ri_2}{\hookrightarrow} R_2$ *with initial pushout (1) for* $K_1 \overset{le_1}{\hookrightarrow} L_1$ *and a span* $s_1 : C_1 \overset{o_1}{\hookleftarrow} S_1 \overset{q_{12}}{\rightarrow} L_2$ *as depicted in Fig. 3, be given.*

1. *Span* s_1 *is called* conflict part candidate *for the pair of rules* (r_1, r_2) *if it satisfies the conflict condition. Graph* S_1 *is called the* conflict graph *of* s_1.
2. *A conflict part candidate* s_1 *for* (r_1, r_2) *is a* conflict part *for* (r_1, r_2) *if* s_1 *fulfills the transformation condition.*
3. *A conflict part candidate* s_1 *for* (r_1, r_2) *is a* conflict atom candidate *for* (r_1, r_2) *if it fulfills the minimality condition w.r.t. the set of all conflict part candidates for* (r_1, r_2).
4. *A conflict atom candidate* s_1 *for* (r_1, r_2) *is a* conflict atom *for* (r_1, r_2) *if* s_1 *fulfills the transformation condition.*
5. *A conflict part* s_1 *for* (r_1, r_2) *is a* conflict reason *for* (r_1, r_2) *if* s_1 *fulfills the completeness condition.*
6. *A conflict reason* s_1 *for* (r_1, r_2) *is* minimal *if it fulfills the minimality condition w.r.t. the set of all conflict reasons for* (r_1, r_2).
7. *Span* $s : L_1 \overset{a_1}{\hookleftarrow} S \overset{b_2}{\rightarrow} L_2$ *is a* conflict reason extension *for* (r_1, r_2) *if it fulfills the extended completeness condition and if there exists a conflict reason* s_1

for (r_1, r_2) with $e' : S_1 \to S$ a so-called embedding morphism being injective such that (5) and (6) in Fig. 4 commute. If the latter is the case, we say that s_1 can be embedded via e' into s.

Note that a conflict part fulfilling the minimality condition is a conflict atom.

Example 2 (Conflict atoms and minimal conflict reasons). Our two example rules in Fig. 2 lead to four pairs of rule combinations to analyze regarding potential conflicts: (*decapsulateAttribute*, *decapsulateAttribute*), (*decapsulateAttribute*, *pullUpEncapsulatedAttribute*), (*pullUpEncapsulatedAttribute*, *decapsulateAttribute*), and (*pullUpEncapsulatedAttribute*, *pullUpEncapsulatedAttribute*). Each pair can be analyzed regarding conflicts that may arise by applying the first rule and in consequence making the second rule inapplicable. To discuss the afore introduced building bricks of conflicts we focus on conflicts that may arise by the rule pair (*decapsulateAttribute*, *pullUpEncapsulatedAttribute*), that means by applying the rule *decapsulateAttribute* and making rule *pullUpEncapsulatedAttribute* inapplicable. Since we do not consider attributes and NACs explicitly in this paper, we neglect them within our conflict analysis. Since these features may restrict rule applications, this decision might lead to an over-approximation of potential conflicts.

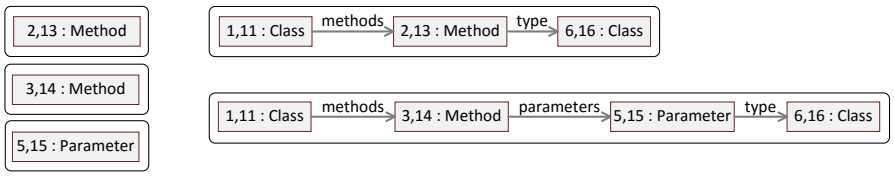


Fig. 5. Conflict atoms (left) and minimal conflict reasons (right) of rule pair (*decapsulateAttribute*, *pullUpEncapsulatedAttribute*)

The root cause of potential conflicts are the three nodes *2:Method*, *3:Method* and *5:Parameter* to be deleted by rule *decapsulateAttribute*. Nodes of the same type are to be used in rule *pullUpEncapsulatedAttribute*. *Method*-nodes are to be deleted twice by rule *decapsulateAttribute* as well as to be used twice in rule *pullUpEncapsulatedAttribute*. Building all combinations this leads to four different conflict atom candidates. Due to the transformation condition, only two of them are conflict atoms: *2,13:Method* and *3,14:Method*, as depicted in Fig. 5 on the left. A further conflict atom is *5,15:Parameter* which is deleted by *decapsulateAttribute* and used by *pullUpEncapsulatedAttribute*. Note that the span notation is rather compact here: Identifying node numbers of rules are used to indicate the mappings of the atom graph into rule graphs. The three conflict atoms are embedded into two *minimal conflict reasons*. Conflict atom *2,13:Method* and the nodes *1,11:Class* and *6,16:Class* are involved within the first *minimal conflict reason*. The remaining two conflict atoms, *3,14:Method* and *5,15:Parameter* can only be covered by a common *minimal conflict reason* due to the completeness condition. This second *minimal conflict reason* also involves nodes *1,11:Class* and *6,16:Class*. These results provide a concise overview on

the root causes of the potential conflicts. The three conflict atoms outline the elements responsible for conflicts and the minimal conflict reasons put them into context to their adjacent nodes.

Remark 1 (conflict reasons for rules). In [12], a conflict reason is defined for a given pair of direct transformations (t_1, t_2). Here, we lift the notion of conflict reason to a given pair of rules and relate it with the notion of conflict part. In fact, the above definition of conflict reason for rules requires that at least one pair of transformations exists with exactly this conflict part as conflict reason. While a pair of conflicting transformations has a unique conflict reason, two rules may be related by multiple conflict reasons. Note, moreover, that our conflict reason notion for rules is not completely analogous to the notion of conflict reason for transformations in [12]. It would be analogous if we considered conflict reasons where both rules are responsible together for delete-use-conflicts. Since such conflict reasons would be constructed from the other ones, and since we aim for compact representations of conflicts, we opted for not including this case separately.

Table 1 provides a conflict notion overview and basic conditions.

basic condition / conflict concept	conflict condition	transf. condition	compl. condition	minimality condition
conflict part candidate	x			
conflict part	x	x		
conflict atom candidate	x			x
conflict atom	x	x		x
conflict reason	x	x	x	
min. conflict reason	x	x	x	x

Table 1. Overview of conflict concepts

3.2 Relations between Conflict Notions of Different Granularities

The subsequent results clarify the main interrelations between the new description forms for conflicting rules. All proofs of new results can be found in the appendix.

In the following extension theorem we state that each conflict part can be extended to a conflict reason. As a special case, it follows automatically that each conflict atom (being a special conflict part) can be extended to a conflict reason.

Theorem 2 (Extension of conflict part to reason). *Given a conflict part* $s_1 : C_1 \overset{o_1}{\hookleftarrow} S_1 \overset{q_{12}}{\rightarrow} L_2$ *for rule pair* $(r_1 : L_1 \overset{le_1}{\hookleftarrow} K_1 \overset{ri_1}{\hookrightarrow} R_1, r_2 : L_2 \overset{le_2}{\hookleftarrow} K_2 \overset{ri_2}{\hookrightarrow} R_2)$, *there is a conflict reason* $s'_1 : C_1 \overset{o'_1}{\hookleftarrow} S'_1 \overset{q'_{12}}{\rightarrow} L_2$ *for* (r_1, r_2) *such that the conflict part* s_1 *can be embedded into it.*

The following lemma gives a more constructive characterization of conflict atom candidates compared to their introduction in Def. 4. This result helps us

to characterize conflict atom candidates for a given pair of rules. Candidates are either nodes deleted by rule r_1 and used by rule r_2 or edges deleted by r_1 and used by r_2 if their incident nodes are preserved by r_1. Edges with at least one incident deleted node are not considered as atom candidates since their deletion is caused by node deletions.

Lemma 1 (Conflict atom candidate characterization). *A conflict atom candidate* $s_1 : C_1 \overset{o_1}{\hookleftarrow} S_1 \overset{q_{12}}{\to} L_2$ *for rules* $(r_1 : L_1 \overset{le_1}{\hookleftarrow} K_1 \overset{ri_1}{\hookrightarrow} R_1, r_2 : L_2 \overset{le_2}{\hookleftarrow} K_2 \overset{ri_2}{\hookrightarrow} R_2)$ *has a conflict graph* S_1 *either consisting of a node* v *s.t.* $o_1(v) \in C_1 \setminus B_1$ *or consisting of an edge* e *with its incident nodes* v_1 *and* v_2 *s.t.* $o_1(e) \in C_1 \setminus B_1$ *and* $o_1(v_1), o_1(v_2) \in B_1$.

Note that, for attributed graphs, the edge in a conflict atom may also be an attribute edge. In this case, the conflict atom would describe an attribute change which is in conflict with an attribute use.

The following corollary allows us to decide that, if there does not exist any conflict atom, there will not be any conflicts at all. This is because, in each conflicting pair of transformations, at least one conflict atom can be embedded.

Corollary 1 (Atoms necessary for conflicts). *Given a conflict reason* $s_1 : C_1 \overset{o_1}{\hookleftarrow} S_1 \overset{q_{12}}{\to} L_2$ *for rules* (r_1, r_2) *then there exists at least one conflict atom for rules* (r_1, r_2) *that can be embedded into* s_1.

The following theorem states that each conflict reason is covered by a unique set of atoms, i.e. all atoms that can be embedded into that conflict reason. With atoms we mean conflict atoms as well as boundary atoms, where a latter one consists merely of a single boundary node. This means that by investigating the set of conflict atoms one gets a complete overview of graph elements that can cause conflicts in a given conflict reason. Moreover, the set of boundary atoms indicates how this conflict reason might be still enlarged with other conflict-inducing edges. Of course, this result also holds for the special case that the conflict reason is minimal.

Definition 5 ((Isolated) Boundary atom). *A span* $s_1^b : C_1 \overset{o_1^b}{\hookleftarrow} S_1^b \overset{q_{12}^b}{\to} L_2$ *is a* boundary part *for rules* (r_1, r_2) *with initial pushout (1) as in Fig. 3 if there is a morphism* $s_B : S_1^b \to B_1$ *such that* $b_1 \circ s_B = o_1^b$ *and* s_1^b *fulfills the transformation condition. A non-empty boundary part* s_1^b *is a* boundary atom *if it fulfills the minimality condition w.r.t. the set of boundary parts for* (r_1, r_2). *Given some conflict reason* $s_1 : C_1 \overset{o_1}{\hookleftarrow} S_1 \overset{q_{12}}{\to} L_2$ *and a boundary atom* s_1^b *with an embedding* $e^b : S_1^b \to S_1$ *into this conflict reason, then* s_1^b *is called* isolated *w.r.t. its embedding* e^b *if there is no conflict atom* $a_1 : C_1 \overset{o_1^a}{\hookleftarrow} A_1 \overset{q_{12}^a}{\to} L_2$ *being embedded into* s_1 *via embedding morphism* $e^a : A_1 \to S_1$ *such that* s_1^b *can be embedded into that conflict atom via* $x : S^b \to A_1$ *with* $e^a \circ x = e^b$.

It is straightforward to show that graph S_1^b of a boundary atom consists of exactly one boundary node being the source or target node of an edge that is potentially conflict-inducing.

Theorem 3 (Covering of conflict reasons by atoms). *Given a conflict reason* $s_1 : C_1 \overset{o_1}{\hookleftarrow} S_1 \overset{q_{12}}{\rightarrow} L_2$ *for rules* (r_1, r_2)*, then the set* A *of all conflict atoms together with the set* A^B *of all boundary atoms that can be embedded into* s_1 covers s_1*, i.e. for each conflict reason* $s'_1 : C_1 \overset{o'_1}{\hookleftarrow} S'_1 \overset{q'_{12}}{\rightarrow} L_2$ *for* (r_1, r_2) *that can be embedded into* s_1 *it holds that, if each atom in* $A \cup A^B$ *can be embedded into* s'_1*, then* s'_1 *is isomorphic to* s_1*.*

Conflict reason extensions contain all graph elements that overlap in a pair of conflicting transformations, even elements that are not deleted and at the same time used by any of the two participating rules. Hence, a conflict reason extension might show too much information. By definition, for each conflict reason extension, there is a conflict reason which can be embedded into this extension. Hence, an extension can always be restricted to a conflict reason. Vice versa, the following theorem shows that each conflict reason (being defined over C_1 and L_2) can be extended to at least one conflict reason extension (being defined over L_1 and L_2).

Theorem 4 (Extension of conflict reason to conflict reason extension). *Given a conflict reason* $s_1 : C_1 \overset{o_1}{\hookleftarrow} S'_1 \overset{q_{12}}{\rightarrow} L_2$ *for rules* (r_1, r_2)*, there exists at least one conflict reason extension* $s : L_1 \overset{a_1}{\hookleftarrow} S \overset{b_2}{\rightarrow} L_2$ *for rules* (r_1, r_2) *such that* s_1 *can be embedded into* s*.*

3.3 Relations of Conflicting Rule Concepts to Critical Pairs

As illustrated in Fig. 1, for each *critical pair*, there exists an essential critical pair that can be embedded into it (see Completeness Theorem 4.1 in [12]). Match pairs of each (essential) critical pair are jointly surjective (according to the minimal context idea). Thus a critical pair might overlap in elements that are just read by both rules and are not boundary nodes, and exactly these overlaps are *unfolded* again in the essential critical pair. This is because the latter overlaps do not contribute to a new kind of conflict. The set of essential critical pairs is thus smaller than the set of critical pairs and, in particular, each essential critical pair is a critical pair (see Fact 3.2 in [12]).

The following two theorems formalize, on the one hand, the relations between conflict reasons for rule pairs as introduced in this paper and essential critical pairs, and on the other hand, the relations between conflict reason extensions and critical pairs. Note that, as explained in Remark 1, there is no 1-1 correspondence of conflict reasons for rules and essential critical pairs, since we abstract from building symmetrical conflict reasons on the rule level for compactness reasons.

Theorem 5 (Essential critical pair and conflict reason). Restriction. *Given an essential critical pair* $(t_1, t_2) = (P_1 \overset{m_1, r_1}{\Longleftarrow} K \overset{m_2, r_2}{\Longrightarrow} P_2)$ *such that* t_1 *causes a delete-use conflict on* t_2 *then the span* $s_1 : C_1 \overset{o_1}{\hookleftarrow} S_1 \overset{q_{12}}{\rightarrow} L_2$ *arising from taking the pullback of* $(m_1 \circ c_1, m_2)$ *is a conflict reason for* (r_1, r_2)*.*

Extension. *Given a conflict reason* $s_1 : C_1 \overset{o_1}{\hookleftarrow} S_1 \overset{q_{12}}{\rightarrow} L_2$ *for rule pair* (r_1, r_2) *then there exists an essential critical pair* $(t_1, t_2) = (P_1 \overset{m_1,r_1}{\Longleftarrow} K \overset{m_2,r_2}{\Longrightarrow} P_2)$ *such that* t_1 *causes a delete-use conflict on* t_2 *with the pullback of* $(m_1 \circ c_1, m_2)$ *being isomorphic to* s_1.

Theorem 6 (Critical pair and conflict reason extension). Restriction. *Given a critical pair* $(t_1, t_2) = (P_1 \overset{m_1,r_1}{\Longleftarrow} K \overset{m_2,r_2}{\Longrightarrow} P_2)$ *such that* t_1 *causes a delete-use conflict on* t_2 *then the span arising from taking the pullback of* (m_1, m_2) *is a conflict reason extension for* (r_1, r_2).

Extension. *Given a conflict reason extension* $s : L_1 \overset{a_1}{\hookleftarrow} S \overset{b_2}{\rightarrow} L_2$ *for* (r_1, r_2) *then the cospan arising from building the pushout of* (a_1, b_2) *defines the matches* (m_1, m_2) *of a critical pair* $(t_1, t_2) = (P_1 \overset{m_1,r_1}{\Longleftarrow} K \overset{m_2,r_2}{\Longrightarrow} P_2)$ *such that* t_1 *causes a delete-use conflict on* t_2.

Bijective correspondence. *The restriction and extension constructions are inverse to each other up to isomorphism.*

Example 3 (Conflict reason extension). Fig. 5 focuses on the conflict atoms and minimal conflict reasons of the rule pair (*decapsulateAttribute*, *pullUpEncapsulatedAttribute*). Fig. 6 relates these new conflict notions with six (out of 15) critical pairs of the considered rule pair. The two *minimal conflict reasons* sufficiently characterize the overlap in the results 3 and 5. Result 1 presents the combination of both *minimal conflict reasons*. Since these results make no use of further overlapping of non-deleting elements they are also conflict reason extensions. Moreover, they correspond to the results of the essential critical pair analysis. *1,11:Class* and *6,16:Class* are two *boundary atoms*. Additional overlapping of the *Attribute*-nodes of both rules in *4,12:Attribute* leads to larger *conflict reason extensions* and to the remaining three results 2,4, and 6. Adding the remaining elements of the LHS of both rules, we obtain a compact representation of all considered critical pairs.

3.4 Characterization of minimal conflict reasons for delete-read conflicts

If rule r_2 is non-deleting, conflicts are merely delete-read conflicts. We show in the following that, in this case, minimal conflict reasons can be characterized by subgraphs of deletion components of rule r_1. In addition, we are able to show a clear hierarchy of conflict notions in the following sense: Conflict atoms cover minimal conflict reasons which cover together with isolated boundary atoms conflict reasons. Since this kind of analysis is simpler and faster, we consider the following overapproximation of conflicting transformation.

If we do not compute an exact overview of all possible delete-use conflicts (as given by regular critical pairs) that a rule may cause on an other rule, but merely compute an over-approximation by concentrating on delete-read conflicts as they occur if we take the non-deleting variant of the original rule r_2, we conjecture a good trade-off between efficiency and precision. First practical tests we have performed are promising.

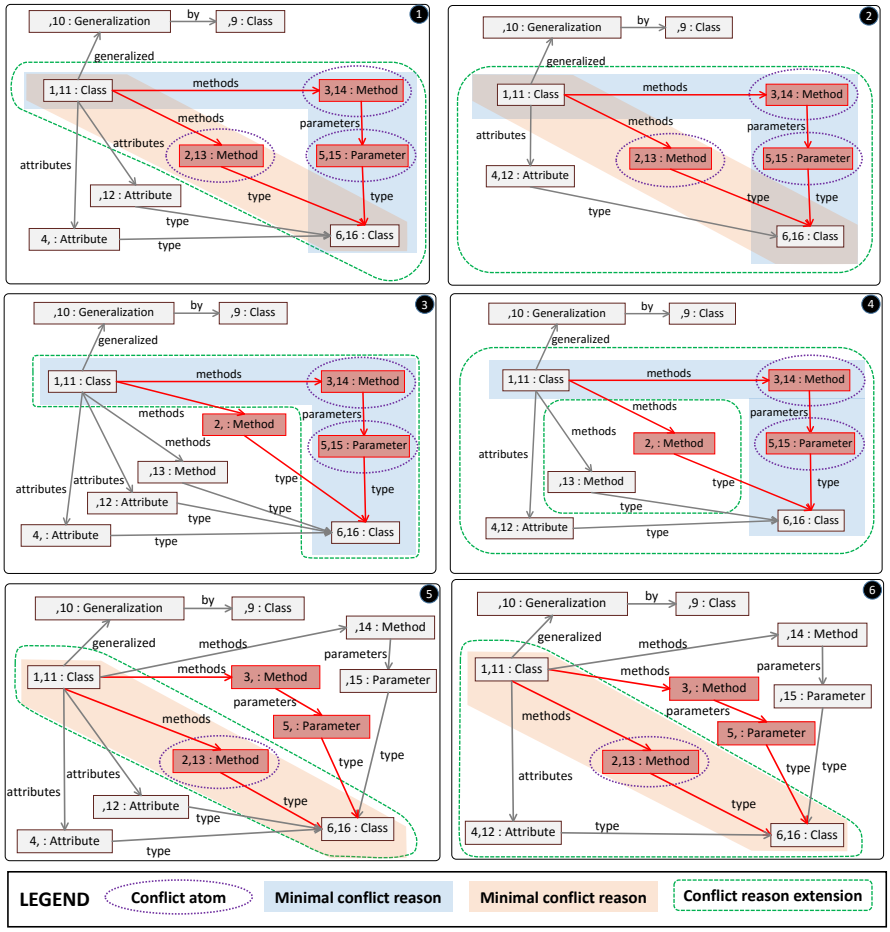


Fig. 6. Representation of six critical pairs arising from the application of rule *decapsulateAttribute* so that rule *pullUpEncapsulatedAttribute* becomes inapplicable; examples of newly introduced conflict notions are indicated.

Definition 6 (Non-deleting rule variant). *Given a graph transformation rule* $r : L \overset{le}{\hookleftarrow} K \overset{ri}{\hookrightarrow} R$, *then* $r' : L \overset{id_L}{\hookleftarrow} L \overset{ri'}{\hookrightarrow} R'$ *is the* non-deleting variant *of* r *with* $ri' : L \hookrightarrow R'$ *being defined by the pushout of* (le, ri).

$$\begin{array}{ccccc} L & \xleftarrow{le} & K & \xrightarrow{ri} & R \\ {\scriptstyle id_L}\downarrow & (=) & \downarrow{\scriptstyle le} & (PO2) & \downarrow \\ L & \xleftarrow[id_L]{} & L & \xrightarrow[ri']{} & R' \end{array}$$

Theorem 7 (Over-approximating delete-use by delete-read conflicts). *Given a pair of rules* (r_1, r_2) *together with the non-deleting variant* r'_2 *of* r_2, *then the following holds: If transformation* t_1 *via* r_1 *causes a delete-use conflict*

on transformation t_2 via r_2 and match m_2, then t_1 causes a delete-read conflict on t'_2 via r'_2 and match m_2.

Proof. This follows directly from Def. 2 since r'_2 the non-deleting variant of r_2 has the same LHS as r_2 and match m_2 of t_2 is identical to the match for t'_2.

This theorem shows that, instead of investigating further all delete-use conflicts that may arise for rules r_1 and r_2, we can also investigate all the delete-read conflicts arising from rules r_1 and r'_2, the non-deleting variant of r_2, without missing any conflicts. However, it may happen that some false positives are computed as the following example illustrates. This is because the opposite direction of the theorem does not hold, i.e. if transformation t_1 causes a delete-read conflict on transformation t'_2 via r'_2, then there does not necessarily exist a transformation t_2 via r_2 and match m_2 because the gluing condition might be violated.

Example 4 (False positive conflict). In the following figure, we see the left-hand parts of rule r_1 that deletes an edge and of rule r_2 that deletes a complete handle. We choose matches such that the handles of both rules overlap completely. While r_1 is applicable, r_2 is not since its match does not fulfill the dangling condition. If we take the non-deleting variant r'_2 of r_2, all its matches fulfill the dangling condition. Using the same matches as before, we get two transformations. They have a delete-read-conflict since the application of r_1 deletes an edge that is used by the application of r_2.

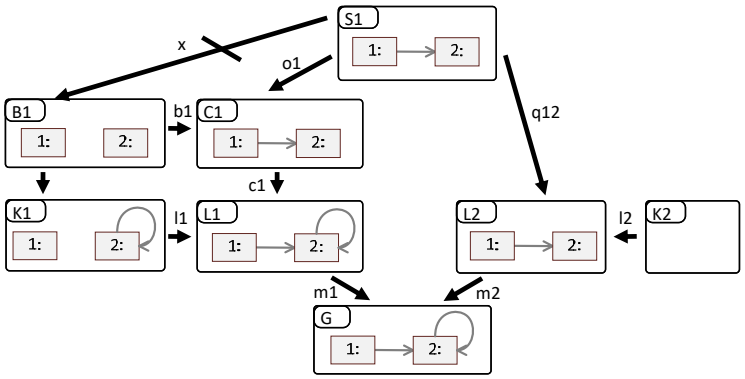


Fig. 7. Example for a false positive conflict

In general, false positives occur if rule r_1 contains some context edge at an overlap node that is deleted by r_2. In addition, r_1 deletes some other item in its overlap with r_2. The application of r_2 would lead to a dangling edge due to the context edge. Hence, there cannot be a transformation t_2 applying r_2 in the described way. The non-deleting variant of r_2, however, would not delete overlap nodes and hence, would fulfill the dangling condition. Therefore, there would be a pair (t_1, t'_2) applying r_1 and r'_2 in the corresponding way and this pair would be conflicting.

In the following, we exploit the fact that rule r_2 is non-deleting.

Lemma 2 (Minimal conflict reason and deletion components). *If a conflict reason $s_1 : C_1 \hookleftarrow S_1 \rightarrow L_2$ for rule $r_1 : L_1 \hookleftarrow K_1 \hookrightarrow R_1$ and non-deleting rule $r_2 : L_2 \hookleftarrow K_2 \hookrightarrow R_2$ is minimal then the conflict graph S_1 is a subgraph of a deletion component of C_1.*

Due to Lemma 2 and Theorem 3 each minimal conflict reason is covered by conflict atoms only (i.e., no isolated boundary atoms). This is because, if isolated boundary atoms would exist, then the minimal conflict reason would be bigger than a deletion component. The minimal conflict reason cannot consist of just one single isolated boundary atom since the conflict condition would not be fulfilled then. Therefore, there has to be another deletion component and this is a contradiction.

The next result states that each conflict atom can be embedded into a minimal conflict reason. This result is closely related with the following one, which shows that each conflict reason can be covered by minimal conflict reasons and isolated boundary atoms. Hence, we have a hierarchical structure of conflict notions: Atoms cover minimal conflict reasons (special case of Theorem 3), which cover together with isolated boundary atoms conflict reasons (Theorem 9).

Theorem 8 (Conflict atom & minimal conflict reason). *Each conflict atom $a_1 : C_1 \overset{o_1}{\hookleftarrow} A_1 \overset{q_{12}}{\rightarrow} L_2$ for rule $r_1 : L_1 \hookleftarrow K_1 \hookrightarrow R_1$ and non-deleting rule $r_2 : L_2 \hookleftarrow K_2 \hookrightarrow R_2$ can be embedded into a minimal conflict reason for (r_1, r_2).*

Theorem 9 (Covering of conflict reasons by minimal conflict reasons). *Given a conflict reason $s_1 : C_1 \overset{o_1}{\hookleftarrow} S_1 \overset{q_{12}}{\rightarrow} L_2$ for rule r_1 and non-deleting rule r_2, then set $M \cup B = \{s_i^m |\ i \in I\} \cup \{s_j^b |\ j \in J\}$ of all minimal conflict reasons and all isolated boundary atoms for (r_1, r_2) that can be embedded into s_1 via a corresponding set of embedding morphisms $E_M = \{e_i |\ i \in I\}$ and $E_B = \{e_{b_j} |\ j \in J\}$* covers *$s_1$, i.e. set $E = E_M \cup E_B$ is jointly surjective.*

Note that, given a graph G, a set of morphisms $E = \{e_i : G_i \rightarrow G | i \in I\}$ is *jointly surjective* if each node and edge in G is in the image of some $e_i \in E$.

Theorem 10 (Essential delete-read critical pair and conflict reason).
Restriction. *Given an essential critical pair $(t_1, t_2) = (P_1 \overset{m_1, r_1}{\Longleftarrow} K \overset{m_2, r_2}{\Longrightarrow} P_2)$ for rule r_1 and non-deleting rule r_2 such that t_1 causes a delete-read conflict on t_2 then the span $s_1 : C_1 \overset{o_1}{\hookleftarrow} S_1 \overset{q_{12}}{\rightarrow} L_2$ arising from taking the pullback of $(m_1 \circ c_1, m_2)$ is a conflict reason for (r_1, r_2).*
Extension. *Given a conflict reason $s_1 : C_1 \overset{o_1}{\hookleftarrow} S_1 \overset{q_{12}}{\rightarrow} L_2$ for rule pair (r_1, r_2) with r_2 non-deleting then there exists an essential critical pair $(t_1, t_2) = (P_1 \overset{m_1, r_1}{\Longleftarrow} K \overset{m_2, r_2}{\Longrightarrow} P_2)$ such that t_1 causes a delete-read conflict on t_2 with the pullback of $(m_1 \circ c_1, m_2)$ being isomorphic to s_1.*
Bijective correspondence. *The extraction and extension constructions are inverse to each other up to isomorphism.*

3.5 Dual Notions for Dependencies

To reason about dependencies of rules and transformations, we consider the dual concepts and results that we get when inverting the left transformation of a conflicting pair. This means that we check if $H_1 \overset{p_1^{-1},m'_1}{\Longleftarrow} G \overset{p_2,m_2}{\Longrightarrow} H_2$ is parallel dependent, which is equivalent to the sequence $G \overset{p_1,m_1}{\Longrightarrow} H_1 \overset{p_2,m_2}{\Longrightarrow} H_2$ being sequentially dependent. This is possible since a transformation is symmetrically defined by two pushouts. They ensure in particular that morphisms $m : L \to G$ as well as $m' : R \to H$ fulfill the gluing condition.

Dependency parts, atoms, reasons, and reason extensions can be defined analogously to Def. 4. They characterize graph elements being produced by the first rule application and used by the second one. Results presented for conflicts above can be formulated and proven for dependencies in an analogous way.

4 Related Work and Conclusion

The critical pair analysis (CPA) has developed into the standard technique for detecting conflicts and dependencies in graph transformation systems [1] at design time. Originally being developed for term and term graph rewriting [15], it extends the theory of graph transformation and, more generally, of $\mathcal{M}$-adhesive transformation systems [16, 2]. The CPA is not only available for plain rules but also for rules with application conditions [17].

In this paper, we lay the basis for a refined analysis of conflicts and dependencies by presenting conflict and dependency notions of different granularity. Furthermore, we investigate their interrelations. The formal consideration shall be used in a new CDA technique where conflict and dependency analysis can go from coarse-grained information about the potential existence of conflicts or dependencies and their main reasons, to fine-grained considerations of conflict and dependency reasons in different settings.

The CPA is offered by the graph transformation tools AGG [18] and Verigraph [19] and the graph-based model transformation tool Henshin [20]. All of them provide the user with a set of (essential) critical pairs for each pair of rules as analysis result. The computation of conflicts and dependencies using the concepted introduced in the present work has been prototypically implemented in Henshin. First tests indicate that our analysis is very fast and yields concise results that are promising to facilitate understandability. However, it is up to future work to further investigate this aspect in a user study.

Currently, we restrict our formal considerations to graphs and graph transformations. Since all main concepts are based on concepts from category theory, our work is prepared to adapt to more sophisticated forms of graphs or graph transformation. Furthermore, it is interesting to adapt the new notions to transformation rules with negative [21] or more complex nested application conditions [17]. Analogously, to handle attributes within conflicts appropriately it is promising to adapt our approach to lazy graph transformations [22] and to come up with a light-weight conflict analysis complementing the work of Deckwerth et al. [23] on conflict detection of edit operations on feature models. They

combine CPA with an SMT solver for an improved handling of conflicts based on attribute changes. Performance is still a limiting factor for applying the CPA to large rule sets. A *family-based analysis* based on the unification of multiple similar rules [24] is a promising idea to save redundant computation effort.

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A Appendix

A.1 Proofs

In the following, the proofs of all the results in Section 3.2 and 3.3 are given.

Theorem 2 (Extension of conflict part to reason). *Given a conflict part* $s_1 : C_1 \overset{o_1}{\hookleftarrow} S_1 \overset{q_{12}}{\rightarrow} L_2$ *for rule pair* $(r_1 : L_1 \hookleftarrow K_1 \hookrightarrow R_1, r_2 : L_2 \hookleftarrow K_2 \hookrightarrow R_2)$, *there is a conflict reason* $s'_1 : C_1 \overset{o'_1}{\hookleftarrow} S'_1 \overset{q'_{12}}{\rightarrow} L_2$ *for* (r_1, r_2) *such that the conflict part* s_1 *can be embedded into it.*

Proof. Due to Def. 4, a conflict part fulfills the transformation condition. Hence, there exist match morphisms $m_1 : L_1 \to G$ for r_1 and $m_2 : L_2 \to G$ for r_2 such that $m_1(c_1(o_1(S_1))) = m_2(q_{12}(S_1))$. Hence, there exists a pair of direct transformations $(t_1, t_2) = (H_1 \overset{m_1,r_1}{\Longleftarrow} G \overset{m_2,r_2}{\Longrightarrow} H_2)$. Consider the pullback of $m_1 \circ c_1$ and m_2 and the corresponding span $s'_1 : C_1 \overset{o'_1}{\hookleftarrow} S'_1 \overset{q'_{12}}{\rightarrow} L_2$. The span s'_1 is a conflict reason for rules (r_1, r_2) according to Def. 4, since it fulfills the conflict condition because of Theorem 1 as well as the transformation and completeness condition (by construction). Because of the pullback property and the fact that $m_1(c_1(o_1(S_1))) = m_2(q_{12}(S_1))$ there exists a unique morphism $e : S_1 \to S'_1$ such that $o'_1 \circ e = o_1$ and $q'_{12} \circ e = q_{12}$. Furthermore, e is injective, since o_1 and o'_1 are injective. Therefore, e is the embedding morphism that embeds the conflict part s_1 into s'_1.

Lemma 1 (Conflict atom candidate characterization). *A conflict atom candidate* $s_1 : C_1 \overset{o_1}{\hookleftarrow} S_1 \overset{q_{12}}{\rightarrow} L_2$ *for rules* $(r_1 : L_1 \overset{le_1}{\hookleftarrow} K_1 \overset{ri_1}{\hookrightarrow} R_1, r_2 : L_2 \overset{le_2}{\hookleftarrow} K_2 \overset{ri_2}{\hookrightarrow} R_2)$ *has a conflict graph* S_1 *either consisting of a node* v *s.t.* $o_1(v) \in C_1 \setminus B_1$ *or consisting of an edge* e *with its incident nodes* v_1 *and* v_2 *s.t.* $o_1(e) \in C_1 \setminus B_1$ *and* $o_1(v_1), o_1(v_2) \in B_1$.

Proof. From the conflict condition it follows: (1) Each edge of graph S_1 is deleted. (2) If some node of S_1 is preserved then this node is incident to a deleted edge in S_1. (3) S_1 consists of at least one graph element that is deleted.

Because of the minimality condition it follows in addition: For each graph S_1 consisting of more than one graph element being deleted, we can find an embedding of a graph S'_1 deleting exactly one graph element. In particular, for an edge e in S_1 with one of its incident nodes n or m being deleted, we have a span with a graph S'_1 consisting merely of one of the nodes n or m being deleted representing an embedding.

Corollary 1 (Atoms necessary for conflicts). *Given a conflict reason* $s_1 : C_1 \overset{o_1}{\hookleftarrow} S_1 \overset{q_{12}}{\rightarrow} L_2$ *for rules* (r_1, r_2) *then there exists at least one conflict atom for rules* (r_1, r_2) *that can be embedded into* s_1.

Proof. A conflict reason fulfills the conflict condition, hence there is at least one graph element x in S_1 that is deleted. Then we have three cases: (1) x is a node, then we consider A_1 to be the graph consisting of this node (2) x is an edge not incident to a deleted node, then we consider A_1 to be the graph consisting of this edge with incident nodes (3) x is an edge incident to a deleted node, then we consider A_1 to be the graph consisting of one of such an incident deleted node.

In all three cases $e : A_1 \to S_1$ is the obvious inclusion. According to Lemma 1 we then have a conflict atom candidate $a_1 : C_1 \overset{o'_1}{\hookleftarrow} A_1 \overset{q'_{12}}{\to} L_2$ with $o'_1 = o_1 \circ e$ and $q'_{12} = q_{12} \circ e$. Obviously, a_1 is embedded into s_1. We still have to show for a_1 that the transformation condition is fulfilled. Since, for the conflict reason s_1, two transformations exist which overlap in at least this conflict reason, and since a_1 is embedded into s_1, the transformation condition for a_1 is fulfilled indeed. Therefore it is a conflict atom.

Theorem 3 (Covering of conflict reasons by atoms). *Given a conflict reason $s_1 : C_1 \overset{o_1}{\hookleftarrow} S_1 \overset{q_{12}}{\to} L_2$ for rules (r_1, r_2), then the set A of all conflict atoms together with the set A^B of all boundary atoms that can be embedded into s_1* covers *s_1, i.e. for each conflict reason $s'_1 : C_1 \overset{o'_1}{\hookleftarrow} S'_1 \overset{q'_{12}}{\to} L_2$ for (r_1, r_2) that can be embedded into s_1 it holds that, if each atom in $A \cup A^B$ can be embedded into s'_1, then s'_1 is isomorphic to s_1.*

Proof. We assume that s'_1 is not isomorphic to s_1: This is possible only if at least one graph element of S_1 is missing in S'_1. This element cannot be a deleted node, a deleted edge with incident nodes being preserved, or a single boundary node, since there are atoms for these cases. Hence, the missing element must be an edge e incident with a deleted node. Since e occurs in S_1, there has to be a corresponding edge e_2 in L_2. Since s'_1 is a conflict reason for (r_1, r_2) as well there exists a pair of conflicting transformations (t'_1, t'_2). The matches of such a pair (t'_1, t'_2) do not identify e with e_2 (because of the completeness condition), but then since t'_1 deletes e, but not e_2 although they have a common incident deleted node, t'_1 cannot be a transformation since e_2 would dangle. Hence, s'_1 cannot be a conflict reason which contradicts our assumption.

Theorem 4 (Extension of conflict reason to conflict reason extension). *Given a conflict reason $s_1 : C_1 \overset{o_1}{\hookleftarrow} S'_1 \overset{q_{12}}{\to} L_2$ for rules (r_1, r_2), there exists at least one conflict reason extension $s : L_1 \overset{a_1}{\hookleftarrow} S \overset{b_2}{\to} L_2$ for rules (r_1, r_2) such that s_1 can be embedded into s.*

Proof. For the conflict reason s_1 there exists a pair of direct transformations $(t_1, t_2) = (H_1 \overset{m_1, r_1}{\Longleftarrow} G \overset{m_2, r_2}{\Longrightarrow} H_2)$ with s_1 being the pullback of $(m_1 \circ c_1, m_2)$. Now we can also build the pullback (PB) of (m_1, m_2) such that we get a conflict reason extension s. In particular, the conflict reason s_1 can be embedded into s because of the pullback property of (PB) and the fact that $m_1 \circ c_1 \circ o_1 = m_2 \circ q_{12}$.

Theorem 5 (Essential critical pair and conflict reason).
Restriction. *Given an essential critical pair $(t_1, t_2) = (P_1 \overset{m_1, r_1}{\Longleftarrow} K \overset{m_2, r_2}{\Longrightarrow} P_2)$ such that t_1 causes a delete-use conflict on t_2 then the span $s_1 : C_1 \overset{o_1}{\hookleftarrow} S_1 \overset{q_{12}}{\to} L_2$ arising from taking the pullback of $(m_1 \circ c_1, m_2)$ is a conflict reason for (r_1, r_2).*
Extension. *Given a conflict reason $s_1 : C_1 \overset{o_1}{\hookleftarrow} S_1 \overset{q_{12}}{\to} L_2$ for rule pair (r_1, r_2) then there exists an essential critical pair $(t_1, t_2) = (P_1 \overset{m_1, r_1}{\Longleftarrow} K \overset{m_2, r_2}{\Longrightarrow} P_2)$ such that t_1 causes a delete-use conflict on t_2 with the pullback of $(m_1 \circ c_1, m_2)$ being isomorphic to s_1.*

Proof. For the restriction case: Since an essential critical pair $(t_1, t_2) = (P_1 \overset{m_1,r_1}{\Longleftarrow} K \overset{m_2,r_2}{\Longrightarrow} P_2)$ is a pair of direct transformations in conflict it follows from Theorem 1 that the conflict condition must be fulfilled for s_1. The completeness condition must be fulfilled because of the definition of an essential critical pair [12].

For the extension case: Since s_1 fulfills the completeness condition there exists a pair of conflicting transformations $(t'_1, t'_2) = (H_1 \overset{m'_1,r_1}{\Longleftarrow} G \overset{m'_2,r_2}{\Longrightarrow} H_2)$ with s_1 arising from a pullback (1) of $(m'_1 \circ c_1, m'_2)$. Because of completeness of critical pairs [2] and completeness of essential critical pairs [12] an essential critical pair $(t_1, t_2) = (P_1 \overset{m_1,r_1}{\Longleftarrow} K \overset{m_2,r_2}{\Longrightarrow} P_2)$ can be embedded into this pair of conflicting transformations via some embedding morphism $m : K \to G$ s.t. $m \circ m_1 = m'_1$ and $m \circ m_2 = m'_2$. Now we show that s_1 is also a pullback of $(m_1 \circ c_1, m_2)$. This follows from the pullback property of the outer pullback (1). Assume a graph X and morphisms $x_1 : X \to C_1$ and $x_2 : X \to L_2$ s.t. $m_1 \circ c_1 \circ x_1 = m_2 \circ x_2$. Then because of the pullback property of the outer pullback (1) and the fact that $m'_1 \circ c_1 \circ x_1 = m \circ m_1 \circ c_1 \circ x_1 = m \circ m_2 \circ x_2 = m'_2 \circ x_2$ it holds that there exists a unique morphism $x : x \to S_1$ s.t. $o_1 \circ x = x_1$ and $q_{12} \circ x = x_2$. Finally, because of Theorem 1 t_1 in the essential critical pair causes a delete-use conflict on t_2.

Theorem 6 (Critical pair and conflict reason extension). Restriction. *Given a critical pair* $(t_1, t_2) = (P_1 \overset{m_1,r_1}{\Longleftarrow} K \overset{m_2,r_2}{\Longrightarrow} P_2)$ *such that* t_1 *causes a delete-use conflict on* t_2 *then the span arising from taking the pullback of* (m_1, m_2) *is a conflict reason extension for* (r_1, r_2).

Extension. *Given a conflict reason extension* $s : L_1 \overset{a_1}{\hookleftarrow} S \overset{b_2}{\to} L_2$ *for* (r_1, r_2) *then the cospan arising from building the pushout of* (a_1, b_2) *defines the matches* (m_1, m_2) *of a critical pair* $(t_1, t_2) = (P_1 \overset{m_1,r_1}{\Longleftarrow} K \overset{m_2,r_2}{\Longrightarrow} P_2)$ *such that* t_1 *causes a delete-use conflict on* t_2.

Bijective correspondence. *The restriction and extension constructions are inverse to each other up to isomorphism.*

Proof. For the restriction case it is obvious that by taking the pullback (1) of (m_1, m_2) the extended completeness condition is fulfilled. Moreover, by taking the pullback of $(m_1 \circ o_1, m_2)$ we obtain a conflict reason for rules (r_1, r_2) that, because of the pullback property of pullback (1), can be embedded into the conflict reason extension.

For the extension case we get by definition of conflict reason extension two transformations $(t'_1, t'_2) = (H_1 \overset{m'_1,r_1}{\Longleftarrow} G \overset{m'_2,r_2}{\Longrightarrow} H_2)$ s.t. s is the span arising from building the pullback (2) of (m'_1, m'_2). By definition of conflict reason extension a conflict reason $s_1 : C_1 \overset{o_1}{\hookleftarrow} S_1 \overset{q_{12}}{\to} L_2$ can be embedded via some embedding morphism $e' : S_1 \to S$ into the conflict reason extension s s.t. $a_1 \circ e' = c_1 \circ o_1$ and $b_2 \circ e' = q_{12}$. Now we can build the pullback (3) of $(m'_1 \circ c_1, m'_2)$ obtaining a span $s'_1 : C_1 \overset{o'_1}{\hookleftarrow} S'_1 \overset{q'_{12}}{\to} L_2$. We show that this new span s'_1 fulfills the conflict condition by contradiction. Assume that s'_1 would not fulfill the conflict condition. Then there exists some injective morphism $x' : S'_1 \to B_1$ s.t. $le_1 \circ x' = o'_1$. Because of the pullback property of (3) and the fact that $m'_1 \circ c_1 \circ o'_1 = m'_2 \circ q'_{12}$ there exists a unique morphism $e_1 : S_1 \to S'_1$ s.t. $o'_1 \circ e_1 = o_1$ and $q'_{12} \circ e_1 = q_{12}$. Furthermore, since o'_1 and o_1 are injective also e_1 is injective. Therefore we can follow that $le_1 \circ x' \circ e_1 = o_1$ s.t. the conflict condition of s_1 would not be fulfilled, which is a

contradiction. We know then because of Theorem 1 that t'_1 causes a delete-use conflict on t'_2. By building now the pushout of (a_1, b_2) we get a pair of jointly surjective morphisms $(m_1 : L_1 \to K, m_2 : L_2 \to K)$ defining the matches of a corresponding critical pair that can be embedded into (t_1, t_2) (see Completeness proof for critical pairs in [2]). Because of the pullback property of (3) the span s'_1 is also a pullback for $(m_1 \circ c_1, m_2)$ s.t. the critical pair is indeed also in delete-use conflict.

The constructions have a bijective correspondence since in the category of typed graphs (or also in a weak adhesive HLR category) a PO over at least one injective morphism (morphism in $\mathcal{M}$, resp.) [2] is also a PB. Moreover, because of Remark 2.25 in [2] the PB in the restriction construction built from two injective, jointly surjective morphisms is also a PO. Moreover, POs and PBs are unique up to isomorphism.

Lemma 2 (Minimal conflict reason and deletion components). *If a conflict reason $s_1 : C_1 \overset{o_1}{\hookleftarrow} S_1 \overset{q_{12}}{\rightarrowtail} L_2$ for rule $r_1 : L_1 \hookleftarrow K_1 \hookrightarrow R_1$ and non-deleting rule $r_2 : L_2 \hookleftarrow K_2 \hookrightarrow R_2$ is minimal then the conflict graph S_1 is a subgraph of a deletion component of C_1.*

Proof. Assuming that the conflict graph S_1 is not a subgraph of one deletion component of C_1, we argue by contradiction and will show that in this case s_1 is not minimal. Since s_1 satisfies the conflict condition, there exists at least one node or edge x in S_1 that is deleted. Now consider a span s'_1 embedded into s_1 such that its conflict graph S'_1 consists of all elements belonging to S_1 that are mapped to the deletion component in C_1 surrounding x. It is obvious that s'_1 is a conflict part candidate. In particular, the conflict condition is trivially fulfilled. We can, moreover, show that s'_1 fulfills the transformation and completeness conditions. This is the case since for s_1 there exists a pair of transformations with valid matches m_1 and m_2 overlapping the deleted elements of r_1 and read elements of r_2 exactly as specified by s_1.

Now there exist also two transformations by restricting the matches m_1 and m_2 to overlap merely in the one deletion component specified by s'_1. We show this by contradiction and by assuming that the accordingly restricted match m'_1 of m_1 would violate the dangling edge condition. This is the case if a node is deleted by r_1 that is matched by m'_1 to a node incident with an edge that is not deleted by r_1. If this node does not belong to the deletion component C_1, then m'_1 matches it to a node that can merely be incident with edges from the LHS of rule r_1, since no overlap on this deletion component takes place. If rule r_1 would not delete one of these incident edges, then it would not be a valid graph rule and this is a contradiction. If on the other hand this node belongs to the deletion component in C_1, then, if m'_1 maps it to a node with dangling edge, then also m_1 would violate the dangling edge condition since m'_1 and m'_2 are equal to m_1 and m_2 on this deletion component and this is a contradiction. Since r_2 is non-deleting, the dangling edge condition for the restricted match of m_2 is obviously fulfilled. Hence, s'_1 is a conflict reason that can be embedded into s_1, and is definitely non-isomorphic to s_1. This is a contradiction to our assumption that s_1 is minimal.

Theorem 8 (Conflict atom & minimal conflict reason). *Each conflict atom $a_1 : C_1 \overset{o_1}{\hookleftarrow} A_1 \overset{q_{12}}{\rightarrowtail} L_2$ for rule $r_1 : L_1 \hookleftarrow K_1 \hookrightarrow R_1$ and non-deleting rule $r_2 : L_2 \hookleftarrow K_2 \hookrightarrow R_2$ can be embedded into a minimal conflict reason for (r_1, r_2).*

Proof. Since a_1 is a conflict atom, there is a pair of conflicting transformations (t_1, t_2) with matches $m_1 : L_1 \to G$ for r_1 and $m_2 : L_2 \to G$ for r_2 such that $m_1(c_1(a_{11}(S_1))) = m_2(a_{12}(S_1))$, i.e, the transformation condition in Def. 4 holds. Hence, by building the pullback of $(m_1 \circ c_1, m_2)$ we get a conflict reason $s_1 : C_1 \overset{o_1}{\hookleftarrow} S_1 \overset{q_{12}}{\to} L_2$ for (r_1, r_2). The conflict condition is fulfilled due to atom a_1; the completeness and transformation condition are fulfilled by construction. Moreover, because of the pullback property a_1 can be embedded into s_1.

We assume that a_1 itself is not isomorphic to s_1 and therefore not a minimal conflict reason yet. Then, starting with the element in A_1, we can enlarge A_1 by successively adding elements of S_1 belonging to the same deletion component of C_1. We stop this addition of elements as soon as the matches arising from m_1 and m_2 by restricting them to overlap merely in the elements in A_1 and added to A_1 fulfill the dangling edge condition. This addition terminates at latest when we have reached all elements present in S_1 surrounding A_1 belonging to the same deletion component of C_1. As argued in the proof of Lemma 2, we can follow that the dangling condition is fulfilled if a complete deletion component is reached as overlap.

The span that arises by this construction fulfills the conflict and completeness condition by construction and it is minimal because we stop adding elements as soon as the transformation condition is fulfilled in addition.

Theorem 9 (Covering of conflict reasons by minimal conflict reasons). *Given a conflict reason $s_1 : C_1 \overset{o_1}{\hookleftarrow} S_1 \overset{q_{12}}{\to} L_2$ for rule r_1 and non-deleting rule r_2, then set $M \cup B = \{s_i^m |\ i \in I\} \cup \{s_j^b |\ j \in J\}$ of all minimal conflict reasons and all isolated boundary atoms for (r_1, r_2) that can be embedded into s_1 via a corresponding set of embedding morphisms $E_M = \{e_i |\ i \in I\}$ and $E_B = \{e_{b_j} |\ j \in J\}$* covers *$s_1$, i.e. set $E = E_M \cup E_B$ is jointly surjective.*

Proof. Assume that E is not jointly surjective. Then there exists at least one graph element x in S_1 that is not in the codomain of one of the morphisms in E.

First assume that x is mapped via o_1 to an edge or a node that is deleted by rule r_1, then x leads to a conflict atom candidate (either with its incident nodes or the node itself) that can be embedded into s_1 (see Lemma 1).

Assume otherwise that x is mapped via o_1 to a node that is not deleted by r_1. Then, either x does not have incident edges to be deleted, i.e., x would represent an isolated boundary node, which would be a contradiction since then x would belong to the codomain of one of the morphisms in E_B, or there is an incident edge y in S_1 that is deleted. Now either both incident nodes of y are preserved such that y with its incident nodes represents a conflict atom candidate, or one of the incident nodes is deleted such that this deleted node represents a conflict atom candidate. So, in all cases, we have found a conflict atom candidate that is embedded into s_1. Therefore, it is a conflict atom in particular.

Because of Theorem 8 we can complete this conflict atom to a minimal conflict reason for (r_1, r_2). In particular, we can add a minimal number of elements of the surrounding deletion component of the atom that are also present in the conflict reason s_1 such that the transformation condition is fulfilled and it can be embedded into s_1. This is a contradiction since now we have that x is in the codomain of one of the morphisms in E_M. Therefore, E is indeed jointly surjective.

Theorem 10 (Essential delete-read critical pair and conflict reason).
Restriction. *Given an essential critical pair* $(t_1, t_2) = (P_1 \overset{m_1,r_1}{\Longleftarrow} K \overset{m_2,r_2}{\Longrightarrow} P_2)$ *for rule* r_1 *and non-deleting rule* r_2 *such that* t_1 *causes a delete-read conflict on* t_2 *then the span* $s_1 : C_1 \overset{o_1}{\hookleftarrow} S_1 \overset{q_{12}}{\rightarrow} L_2$ *arising from taking the pullback of* $(m_1 \circ c_1, m_2)$ *is a conflict reason for* (r_1, r_2).
Extension. *Given a conflict reason* $s_1 : C_1 \overset{o_1}{\hookleftarrow} S_1 \overset{q_{12}}{\rightarrow} L_2$ *for rule pair* (r_1, r_2) *with* r_2 *non-deleting then there exists an essential critical pair* $(t_1, t_2) = (P_1 \overset{m_1,r_1}{\Longleftarrow} K \overset{m_2,r_2}{\Longrightarrow} P_2)$ *such that* t_1 *causes a delete-read conflict on* t_2 *with the pullback of* $(m_1 \circ c_1, m_2)$ *being isomorphic to* s_1.
Bijective correspondence. *The extraction and extension constructions are inverse to each other up to isomorphism.*

Proof. The restriction and extension case follow as a special case from Theorem 5.

Because of Theorem 4.2 in [12] each essential critical pair is unique w.r.t. its conflict reason. A conflict reason for a pair of transformations as introduced in [12] is in particular a conflict reason for a pair of rules if one of the rules is non-deleting (see Remark 1), since we can then safely abstract from the symmetrical case. Therefore each essential critical pair $(t_1, t_2) = (P_1 \overset{m_1,r_1}{\Longleftarrow} K \overset{m_2,r_2}{\Longrightarrow} P_2)$ such that t_1 causes a delete-read conflict on t_2 is unique w.r.t. the corresponding conflict reason for the rule pair (r_1, r_2).